Convergence in the World Economy – Evidence from the Last Fifty Years

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Inequality and Income Convergence in the World Economy – Evidence from the Last Fifty Years

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Abstract

Convergence in incomes, productivity-catch-up and technology gaps have been studied extensively. This study focuses on convergence in income and income distributions, two factors that determine welfare. The paper offers an intuitive notion of income convergence which is then used in establishing an analytical link between income (sigma) convergence and inequality measures. Empirical results reported are based on data series available from UQICD V3.0 (University of Queensland International Comparison Database) covering 185 countries and the period 1970 to 2019 covering pre- and post-globalization years. Using recently developed econometric methods, the paper finds strong evidence of weak-sigma convergence, absolute and conditional β convergence; as well as convergence tested using economic transition curves proposed in Phillips and Sul (2009). However, the results vary depending on the groups of countries considered with robust results for the group of countries classified as the upper-middle income group. World inequality, accounting for income distributions within countries, peaked during 1990 to 1995 with a Gini coefficient around 0.72 decreasing to 0.575 by 2019. There is evidence of a reduction in between-country inequality coupled with a rise in within-country inequality. The paper proposes a new entropy-based measure of divergence between income distributions. Under Pareto-lognormal specification, fitted income distributions for a large number of countries for years between 1970 and 2019, available from the UQICD database, show a significant reduction in divergence in income distribution of countries in the world from 1985 to around 2015 but increasingly slightly until 2019.

Convergence in the World Economy – Evidence from the Last Fifty Years

1. Introduction

Convergence in the world economy is a topic with considerable scope. Solow's (1956) growth model provided a framework for studying catch-up and convergence in incomes (Baumol, 1986; Barro and Sala-i-Martin, 1992; Sala-i-Martin, 1996; Phillips and Sul, 2009; Kong etal, 2019)), price level convergence (Rogoff, 1996; Glick, 2007; Romero, et al 2020) productivity catch-up and convergence (Griffith et al, 2004; Andrews and Westmore, 2014; Johansson et al 2013; Rodrik, 2013; and Guillemette et al, 2017) , technology transfers, convergence in factor remunerations, factor shares not to mention convergence in institutional, organizational and managerial structures across countries. As these convergence studies have the primary goal to understand the assess the potential for growth in incomes, the main focus of this study is to examine convergence in incomes in the world economy in the last half century.

Growth in per capita real incomes, which are adjusted for price level differences across countries and movements in prices over time, are used in examining catch-up and convergence in incomes across countries, an approach underscored in studies led by Barro and Sala-i-Martin (1992). However, use of per capita incomes in countries is likely to overstate convergence of incomes of individuals across countries as it ignores differences and temporal changes in the distribution of incomes within countries. Individuals' levels of income and inequality in the distribution of incomes of individuals determine the economic welfare in a society. In a path-breaking contribution, Sen (1976) established an analytical framework for welfare comparisons across two communities that are not necessarily of same size but consuming quantities of same types of commodities. The main message from Sen (1976) is that welfare comparisons can be reduced to two measurable components: average incomes in the societies which are adjusted for price level differences, and the Gini measures of inequality in the distribution of incomes in these societies. The result implies that per capita real income adjusted by the factor (1-Gini) to account for disparities in the distribution of income provides a measure of welfare which can be used for welfare rankings.

As there is a vast literature on income convergence and the factors, including globalization, explaining the presence or lack of convergence in selected country groups, this study is more conceptual and methodological focusing on measurement and testing procedures and report empirical findings on convergence over the last fifty years. The paper introduces and makes use of a unique data set that has recently been made available through UQICD V 3.0 (University of Queensland International Comparisons Database). UQICD provides data on constant price real per capita incomes for 185 countries after adjusting for differences in currency denominations and in relative price levels across countries and price movements over time. This data set also provides four selected fitted parametric distributions for 159 countries for the period 1970 to 2019.

The paper starts, in section 2, with an intuitive notion of income convergence across countries and articulates different types of income convergence studied in the paper, thereby establishing a link between income convergence and global inequality. Section 3 introduces the reader to the unique and recently available data source, UQICD V3.0, used in the study. Section 4 presents several convergence-related stylized facts covering the last fifty years. Measures of global growth and inflation over the study period along with profiles of growth for geographical regions as well as income groupings are presented. Growth incidence curves for the whole fifty-year period and for sub-periods are presented. Section 5 presents our findings on beta- and sigma-convergence using recently developed econometric tools.

Section 6 breaks new ground as it develops a statistical tool to study convergence in income distributions. A new measure of divergence between two income distributions and a measure for distributions for countries within a group. These measures are used in examining convergence in income distributions and inequality. The paper concludes with a few remarks in Section 7.

2. Convergence and Income-Population Contribution Ratio

We introduce an intuitive notion of income convergence which forms the basis for a formal test procedure used in Section 5. Income convergence across countries can be evidenced if each country contributes to world income in proportion of its population, or simply convergence is attained when the share of the country in world income equals its population share. This means that the population-income contribution ratio converges to 1 overtime.

To state formally, let per capita income and population of country *i* in period *t* are, respectively, denoted by $\bar{x}_{it}$ and N_{it}. Then the income share, and population share of country *i* in the world are given by:

$$IS_{it} = \frac{N_{it} \cdot \bar{x}_{it}}{\sum_{j=1}^{M} N_{jt} \cdot \bar{x}_{jt}} \quad and \quad PS_{it} = \frac{N_{it}}{\sum_{j=1}^{M} N_{jt}}$$

Then income convergence occurs if

$$IPCR_{it} = \frac{IS_{it}}{PS_{it}} = \frac{\dfrac{N_{it} \cdot \bar{x}_{it}}{\sum_{j=1}^{M} N_{jt} \cdot \bar{x}_{jt}}}{\dfrac{N_{it}}{\sum_{j=1}^{M} N_{jt}}} = \frac{\bar{x}_{it}}{\sum_{j=1}^{M} N_{jt} \cdot \bar{x}_{jt} \Big/ \sum_{j=1}^{M} N_{jt}} = \frac{\bar{x}_{it}}{\sum_{j=1}^{M} PS_{jt} \cdot \bar{x}_{jt}} = \frac{\bar{x}_{it}}{\bar{x}_{Wt}}$$

where $\bar{x}_{Wt}$ is the *world per capita income* and *M* is the number of countries.

Therefore, convergence occurs if

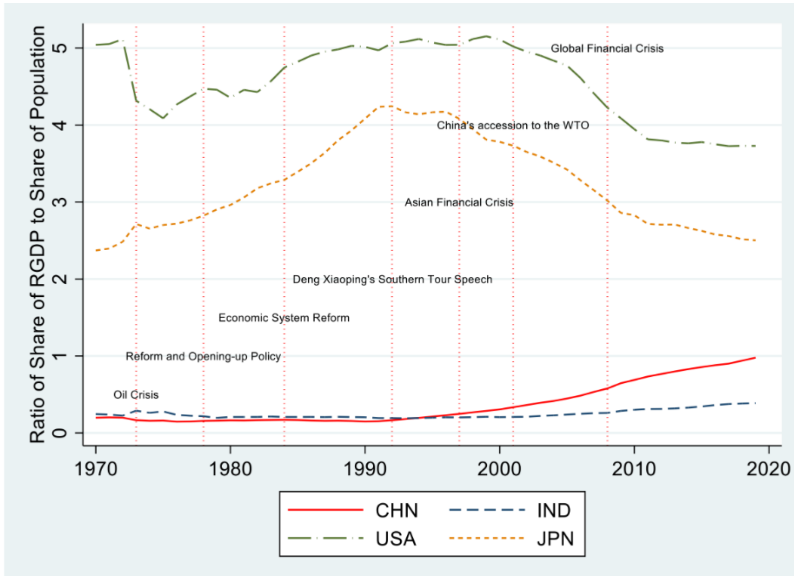
$$\lim_{t \to \infty} IPCR_{it} \to 1 \tag{1}$$

Or, equivalently, income convergence is evident if the ratio per capita incomes of countries in the world converge towards world per capita income. The IPCR is like the notion of *relative transition coefficient* introduced in Phillips and Sul (2009). Their coefficient, defined in Section 6, uses logarithms of incomes instead of incomes here.

Figure below shows the IPCR profiles of USA, Japan, China and India. The figure shows considerable disparities in this ratio in the early period. Since the 1990's, IPCRs for USA and Japan are declining whereas the ratio for China and India are moving upwards with the ratio for China close to 1.

Figure 1: Share in World GDP relative to Share in Population

(selected countries: 1970 to 2019)



Source: authors' calculations based on data from UQICD V 3.0

3. Alternative concepts of Convergence

Income convergence refers to the phenomenon wherein per capita incomes in individuals in a country or per capita incomes across countries converge to the same level. The following notions of convergence are examined in the paper.

Convergence of incomes of people in the world as a whole: Here all the people of the world[1] are considered as a single group and the question is whether incomes of these individuals converge over time.

Let $\{x_{it}:\ i=1,2,...,N\ \ and\ t=1,2,...T\}$ represent incomes of individuals (could be per capita incomes in different countries) over period t. Incomes are assumed to be strictly positive.

We start with the notion of convergence between two income streams which states that

$$\lim_{t\to\infty}\frac{x_{it}}{x_{jt}}=1. \tag{2}$$

Definition: Sequence of incomes of *N* individuals, $\{x_{it}:\ i=1,2,...,N\ \ and\ t=1,2,...T\}$, is said to be convergent if for any pair of individuals *j* and *i* , their ratio of incomes converges to 1 as in equation (1).

Result: Income sequence of *N* individuals is convergent according to above definition *if and only if*, for every individual *j*, the ratio of individual income to average income converges to 1. That is

[1] Instead of the whole world, one may restrict analysis to individuals within a country or within a region.

$$\left\{\lim_{t\to\infty}\frac{x_{it}}{x_{jt}}\to 1\ \forall pairs\ of\ \ individuals\ i\, and\ j\right\}\Leftrightarrow\left\{\lim_{t\to\infty}\frac{x_{jt}}{\sum_{i=1}^{N}x_{it}\Big/N}\to 1\ \forall\ i=1,2,...,N\right\}$$

Proof: If x's are sequences of strict positive real numbers, proof of this is straight forward.

This basically means all incomes converge to the average income enjoyed by the group of individuals. This means in the long run all the individual incomes tend to the same level which is the per capita income of the group. An implication of this convergence is that inequality in incomes among the individuals decline over time and eventually converges to zero.

Corollary 1: A corollary of this result is that, for any given very small positive ε there exists a T such that after that period difference between x_{it} *and* $\bar{x}_t$ is less than ε . This means that incomes of all the individuals would be very close to the mean.

Corollary 2: A further implication of this is that convergence in incomes according to definition above is that inequality in the distribution of incomes goes to zero.

Convergence between countries: Given that population sizes change over time and that it is difficult to observe incomes of all the people in all the countries, convergence between countries is notionally easier to study in practice. The notion of convergence is the same as the one above except that convergence is defined using per capita incomes in each country. Let *M* denote the number of countries. Convergence is said to occur if, per capita income of country *i* in time period *t* is denoted by $\bar{x}_{it}$, the following ratio

$$IPCR_{it}=\frac{\bar{x}_{it}}{(1/M)\sum_{i=1}^{M}\bar{x}_{it}}\to 1\ as\ t\to\infty.$$

This measure is similar to that used by Phillips and Sul (2009) except that they use log's of per capita incomes in their analysis. As countries tend to be of different of population sizes, a more meaningful convergence measure would use a ratio relative to population weighted average of per capita incomes from all the countries. That is to use:

$$ICPR_{it}=\frac{\bar{x}_{it}}{\sum_{i=1}^{M}w_{it}\bar{x}_{it}}\to 1\ as\ t\to\infty$$

where w_{it} is share of population of country i in the world.

Convergence in within-country inequality over time: Here the notion under consideration is whether inequality measures in different countries are converging over time.

Convergence in income distributions over time: Although the main objective is to study the convergence of income distribution, there are different types of distributional change questions that are relevant. Consider a population that consists of M subpopulations over time. The population could be a region, with subpopulations being the countries within that region, or it could be a country, with subpopulations being its provinces. Understanding the dynamics of income distribution within and across these subpopulations involves addressing several key questions:

- *How has the distribution of income changed within a population or subpopulation over time*? This question focuses on temporal changes in income distribution within a single population. It aims to identify trends, such as increasing inequality or improved income equality, over a specified period. Methodology to answer this is a question was developed and implemented in Hajargasht (2024).

• *How different is income distribution between a set of subpopulations at a particular time*? This question compares income distributions across different subpopulations at a single point in time. The goal is to understand the disparities or similarities between subpopulations. This involves comparisons of income distributions for every pair of subpopulations and then obtain an overall measure.

• *How has the difference in income distribution among subpopulations evolved over time*? This question explores the temporal evolution of income distribution disparities between subpopulations. It seeks to determine whether subpopulations are converging or diverging in terms of income distribution.

• *How has the distribution of income changed across the entire population, particularly in relation to the changes within subpopulations?* This question looks at the overall population's income distribution dynamics while considering the contributions and changes within individual subpopulations. It assesses how shifts within subpopulations aggregate to impact the entire population's income distribution. Analysis may include decomposing overall inequality into within-subpopulation and between-subpopulation components.

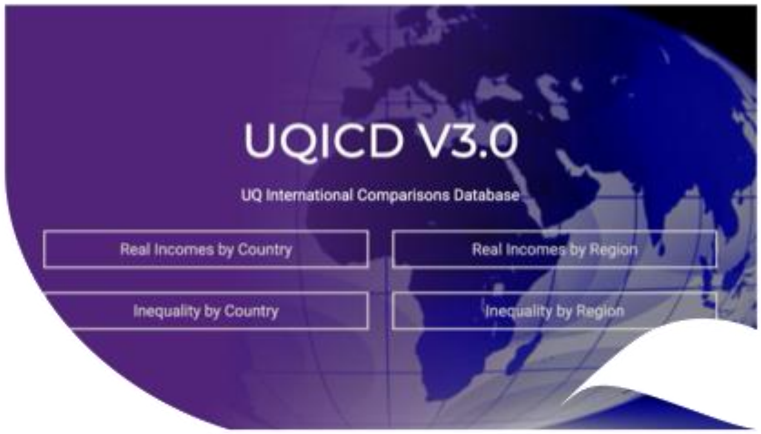
This paper reports results concerning different types of divergence discussed in this section. Such testing is possible as UQICD offers fitted parametric distributions for all the countries covered in the database.

4. Data Source: UQICD V3.0

All the empirical results reported in this paper are based on data drawn from UQICD (University of Queensland International Comparisons Database) V3.0 released in March 2023. The UQICD is the only database of its kind which combines and provides series on purchasing power parities (PPPs) of currencies, real incomes and inequality. As mentioned in the introduction, real incomes and inequality are the key drives of economic welfare. The UQICD V3.0 is designed to complement internationally comparable price and income series available through the Penn World Table (PWT). It also complements several income distribution databases currently available – World Inequality Database (WID); Income Distribution Database, OECD; Standardized World Income Inequality Database (SWIID); and the World Income Inequality Database (WIID).

An additional feature that distinguishes UQICD from other databases is that real income and inequality series are provided for: (i) geographical regions of the world; (ii) for groups of countries based on levels of per capita income as classified by the World Bank (2020); and (iii) the administrative regions OECD and EU. *Recently, an additional use-friendly feature was added to UQICD whereby users can construct real income, growth and inflation for any groups of countries they may choose.*

Data series can be accessed using the URL: https://uqicd.economics.uq.ed.au which leads to the following user-friendly interface:



Details of the methodology used in the compilation of the series available through UQ can be found in: Rao, D.S. Prasada, Alicia Rambaldi, Gholamreza Hajargasht and William E Griffiths (2023): *The University of Queensland International Comparison Database Version 3.0: User Guide*, CPA Working Paper Series No. WP09/2022 revised in 2023.

Coverage of UQICD: Data series are available for 185 countries and the period 1970 to 2019.

UQICD V3.0 has four major modules. Data series available under each of the modules are listed below.

Module 1: Real Incomes by Country

This module provides series on: (i) *Purchasing Power Parities (PPPs)* at the GDP level as well as for Consumption, Government and Gross Capital Formation; (ii) *Per Capita Real Expenditure Series by country (current prices)* – RGDP - these series are obtained by converting expenditures in different countries in any given year using PPPs – these are also referred to as *current price* real expenditure series as the PPPs of the year are used to convert expenditures; (iii) *Per Capita Nominal Expenditure series by country* – NGDP - these series are obtained by using market exchange rates to convert expenditures in different countries into US dollars; and, finally, (iv) *Constant Price Real Expenditure Series by country (constant 2011 prices)* – *CRGDP*- all the series here are adjusted for price level differences across countries using PPPs and for price changes over time and are expressed in 2011 prices.

Module 2: Inequality and Income Distributions by Country

The UQICD provides panels of income distributions constructed using methodology developed by the authors and explained in detail in the User Guide for UQICD (Rao et al, 2023). Parametric income distributions are fitted to income share data from World Bank's POVCAL database. Estimated parameters and their standard errors are available for: Lognormal Distribution; Pareto-lognormal distribution; Generalized Beta Distribution of the second kind (GB2); and Mixtures of lognormal distributions. For all these distributions UQICD provides: (i) Estimated parameters and their standard errors; (ii) estimated mean and median income; (iii) Gini and Theil measures of inequality; and (iv) shares of bottom 10 and 30%, of top 10 and 30% of population.

Module 3: Real incomes by regions

This model is unique and the only one of its kind available globally. Series are available for: geographic regions; income-level based groupings of countries; administrative groups consisting of OECD and EU.

The following basic series are available in this module: POPULATION: Total population by region; RGDP: Real GDP of the region: sum of real GDP of all the countries belonging to the region (expressed in US dollars; converted using PPPs); NRGDP: nominal GDP of the region: sum of nominal GDP of all the countries belonging to the region (expressed in US dollars; converted Exchange Rates); and CRGDP: Constant real GDP of the region in Year 2011 prices: sum of constant real GDP in 2011 prices of all the countries belonging to the region (US dollars, POPP terms)

Decomposition of change in Real GDP of a given region over time

The UQICD provides decomposition of changes in real GDP of a region over time into three components: Domestic Growth; Domestic Inflation; PPP effects; and Price effects (domestic inflation and PPP effects combined). Decompositions provided in UQICD are compiled using methodology provided in Balk, Rambaldi and Rao (2022). The series available are: (i) RGDP_GR: The change in Real GDP for the countries in the region; (ii) DOM_GR: Weighted average of domestic growth rates of countries in the region; (iii) DOM_INF: Weighted average of domestic inflation rates of countries in the region; (iv) PPP_EFFECT: Effect of changes in PPPs over time on changes in real GDP of the region; and (iv) PRICE_EFFECT: Total contribution of price related effects - product of DOM_INF and PPP_EFFECT.

Module 4: Inequality by Region

Under this module in UQICD, various inequality measures are provided for the regional groupings described before. These are computed by constructing regional income distributions as mixtures or weighted averages of distributions of countries belonging to the region. Hence, inequality data will depend upon the user’s choice from the four distributions, viz., Log-normal distribution; Pareto-lognormal distribution; Generalized beta of the second kind (GB2) distribution; and Mixture of lognormal distributions

Once the user chooses one of the distributions the following measures can be downloaded.POPULATION: Total population of the region; RGDP: Real GDP of the region in PPP terms and current prices; estimated Gini and Theil inequality measures; Estimated shares of the poorest 10 and 30% of the population and for the richest 1 and 10% of the population

5. Growth, Income Convergence and Inequality – Preliminary observations from UQICD Data Series

5.1 World economy over the last fifty years – some stylized facts

Data drawn from Module 3 of UQICD provides estimates of population and size of the world economy in nominal (NRGDP), real (RGDP), and constant price real terms (CRGDP). The world population has doubled during the fifty-year period, increased from 3.374 to 7.619 billion. During this period, the exchange rate converted GDP of the world increased from US$2.931 trillion to $86.473 trillion. However, PPP converted real GDP (adjusted price level differences between countries) increased from $3.428 trillion to $133.088 trillion, a 38.8-fold increase. However, part of this change could be due to price changes over time. Size of the world economy adjusted for price differences across countries and price changes over time is measured by CRGDP increased from $20.397 trillion to 118.187 trillion, a 5.8-fold increase over the period. However, growth in the world economy has not been uniform across the five decades. These are shown in the table below.

Table 1: Size of the World Economy: Last Five decades

	WORLD				
	1970	1980	1990	2000	2019
Population (bill)	3.374	4.096	5.263	6.069	7.619
RGDP ($ trillion)	3.428	11.715	24.823	43.222	133.088
NGDP ($ trillion)	2.931	11.218	22.743	33.166	86.473
CRGDP ($trillion)	20.397	30.495	45.893	61.163	118.187

Source: UQICD V3.0, Module 3

The UQICD provides a decomposition of growth in the real size of the world economy into three components: domestic growth; domestic inflation; PPP effect; and Price Effect which is the product of Inflation and PPP effects which are both driven by changes in prices over time. This decomposition from Balk, Rambaldi and Rao (2022) is shown below:

$$\frac{RGDP_{W,t}}{RGDP_{W,t-1}} = \frac{\sum_{j=1}^{M} RGDP_{j,t}}{\sum_{j=1}^{M} RGDP_{j,t-1}} = \prod_{j=1}^{M}\left[\frac{CGDP_{j,t}}{CGDP_{j,t-1}}\right]^{\omega_j} \prod_{j=1}^{M}\left[\frac{Def_{j,b,t}}{Def_{j,b,t-1}}\right]^{\omega_j} \prod_{j=1}^{M}\left[\frac{PPP_{j,t-1}}{PPP_{j,t}}\right]^{\omega_j}$$

$$= \prod_{j=1}^{M}\left[Dom.GR_{j,t-1,t}\right]^{\omega_j} \prod_{j=1}^{M}\left[Dom.Inf_{j,t-t,t}\right]^{\omega_j} \prod_{j=1}^{M}\left[PPP\,effect\right]^{\omega_j}$$

$$= Global\ growth \times Average\ Dom\ Inf \times PPP\,change\,effect$$

$$= Global\ growth \times Global\ Inflation$$

where the weights w_j are Sato-vartia weights which in turn depend on shares of countries in the world GDP in the first and last periods. Results from the decomposition are shown below:

Table 2: Decomposition of Change in World Real GDP over the period 1970 – 2019

Measure	VALUE
RGDP_1970	$3.428 trillion
RGDP_2019	$125.929 trillion
RGDP_2019/RGDP_1970	36.73
DOM_GR	6.33
DOM_INF	61.64
PPP_EFFECT	0.10
PRICE_EFFECT	6.13

Notes: components are identified using the same set of countries in both years

Table 2 shows that RGDP in 2019 is roughly 36 times that of RGDP in 1970. Of this change domestic growth, computed as Sato-Vartia weighted average of growth rates in all the countries included in the computation, accounts for 6 times initial the value. The total price effect (inflation and PPP effect) contributes a similar amount.

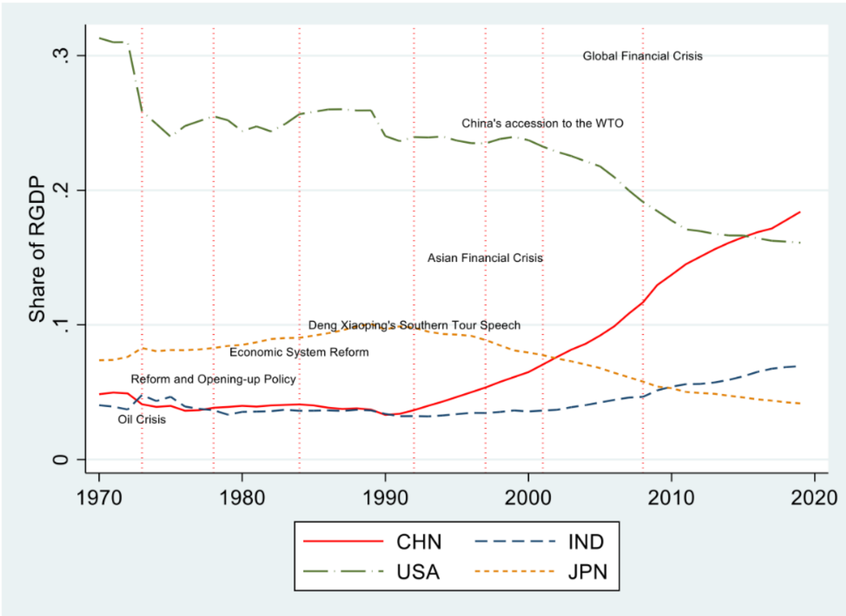
Table 3: Contribution of selected countries to Global growth

Ln(Growth rates)		% contribution
LN(RGDP ratio)	3.60	
LN (DOM_GR)	1.79	49.68%
Australia	0.02	0.98%
China	0.45	25.36%
Germany	0.05	2.62%
India	0.15	8.37%
Japan	0.07	3.90%
United States	0.33	18.51%

Table 3 shows that contribution of domestic growth across countries roughly fifty percent of the change in the size of the world economy over the fifty-year period. Surprisingly, China's contribution to growth is around 25% compared to 18.5% from USA, 8.37% from India and 3.90% from Japan.

Figure 2 shows contributions of USA, Japan, China and India to world growth over ten-year sub-periods starting from 1970. Contributions of both China and India were low at the beginning of the period but China's contribution has overtaken Japan's contribution in 2000 whereas Indian contribution is higher than that of Japan since 2011.

Figure 2: Contributions to Global Growth
(by decades and for selected countries)



Source: authors' calculation using series from UQICD V 3.0

These tables show that contribution of low-income countries in early 1970s has been steadily increasing over time while contributions of USA and Japan have been declining over the same period.

There are strong signals and evidence of convergence in incomes from UQICD data. Table 4 shows the shares of income-based groups of countries over time.

Table 4: Shares in world Real GDP

Countries by income Groups	1970	2000	2019
Low income	1	1	1
Lower middle income	9	9	15
Upper middle income	16	21	35
High income	74	69	49

Notes: Country groups are based on World Bank classification of countries by real incomes in 2017. For example, some countries in upper middle income in 2017 could belong to low-income countries.

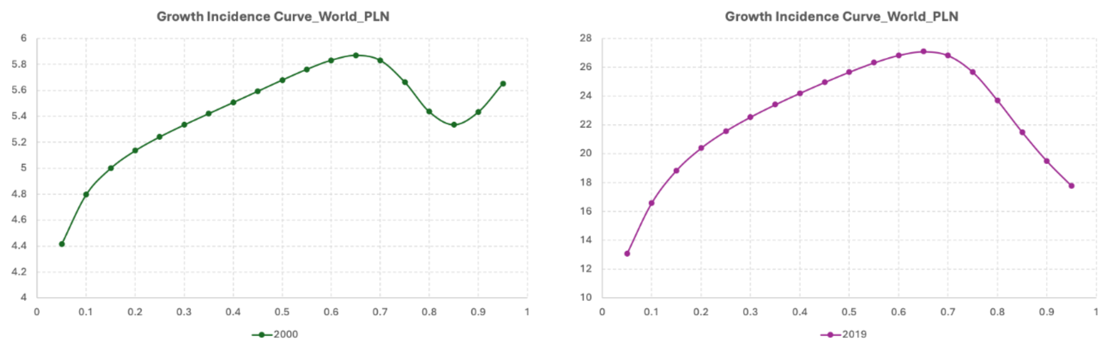
There is an appreciable decline in the share of high-income countries over the period 2000 to 2019 with corresponding increases in shares of upper and lower middle-income groupings. These are the groups dominated by China and India and other emerging economies. The enormous change in shares can be seen from growth incidence curves for the period 1970 to 2000 and 1970 to 2019.

5.2 Growth Incidence Curves – puzzle of vanishing Elephant Curve

Growth incidence curves shown below are derived using the Pareto-lognormal parametric specification for income distributions of the countries. Global income distributions in any given year are population weighted mixture of distributions of all the countries. Further these distributions are all fitted to incomes expressed in constant real incomes (in 2011 prices) and thus be used in computing the necessary growth rates.

We calculate the growth incidence curve using UQICD estimates of the population share-weighted mixture of Pareto-lognormal for global income distribution. Growth for the *i-th* percentile at time t with respect to time t_0 can be defined as $GI_i = \frac{y_t^i - y_{t_0}^i}{y_{t_0}^i}$ with $y_t^i = m_t L_t\left(p^i\right)$ where m_t is the world's mean of income at time t and the associated Lorentz curve calculated at *i-th* percentile p^i. UQICD provides the world distribution of income for several parametric distributions including Pareto-lognormal and from that we can obtain the associated Lorentz curve.

Figure 3: Growth Incidence Curves: 1970 to 2000 and 1970 to 2019



Caution needs to be exercised in looking at these curves simultaneously as the vertical axis scaling is different[2]. The curve on the left for the period 1970 to 2000 resembles the famous *elephant curve* first depicted in Lakner and Milanovic (2013). It is clear from these two graphs that poorer sections of the community enjoyed low growth rates compared to high growth rates enjoyed by upper middle and richer sections. These growth rates, especially in the graph to 2019, are largely influenced by growth performance of China and India. Compared to Lakner and Milanovic graph, our graphs are less granular at the top end of the distribution. Our fitted distributions are based on World Bank's POVCAL data which are based on survey data, and it is possible that incomes for the top 5 and 1% are likely to be under reported.

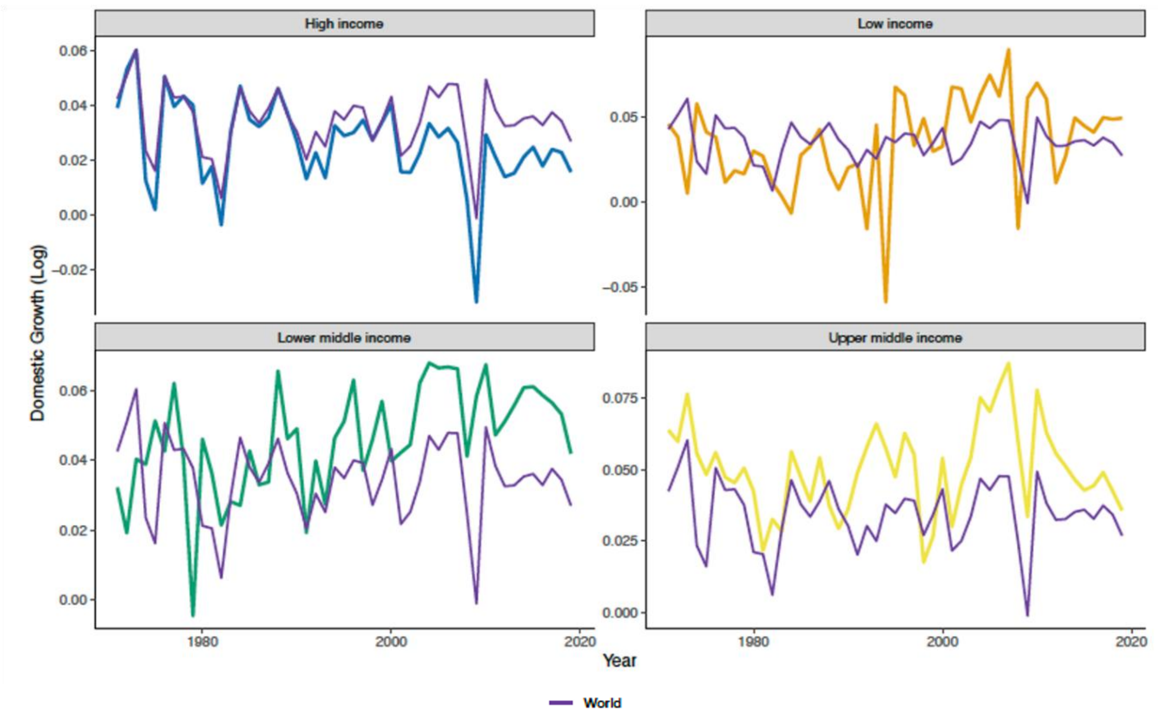
These graphs provide sufficient evidence in support of the huge shift in shares from rich country-group to upper middle and lower-middle class shown in Table 4. What is puzzling in the incidence curves above is that the elephant curve shape evident in the curve for the period 1970 to 2000 has completely vanished with the addition of income distribution data for two additional decades. We are currently investigating the puzzle and to see if we can identify the sources of this puzzle and to see if we can pin-point a year that shows a significant change in shape of the growth incidence curve.

5.3 Growth performance of income-based groups of countries

Further evidence that supports the notion of catch-up and convergence can be seen from the growth performance of income-based groups of countries. The following figure shows regional growth rates for low, lower-middle, upper-middle and high-income country groups from Module 4 of UQICD. These growth rates are presented at five-year intervals.

[2] The curves shown above are constructed using current price income series. Curves based on constant price series are more appropriate for the purpose of analysis. We are in the process of revising these figures which will be incorporated in the next version of the paper.

Figure 4: Growth-rates by Income-based Groups of Countries
(five-year intervals from 1970)



The figure shows that high income countries had growth rates very similar to those of the world, but from year 2000 onwards their growth rates are well below the world growth rates. In contrast growth rates of lower-middle income and upper-middle income country groups have growth rates well above those of the world, especially from year 2000. Low-income countries in contrast had growth rates well below the world until recently. Over the last ten years, several African countries posted impressive growth rates. The dominant performance of upper and lower-middle income countries, again largely driven by growth rates in China and India, has implications for any analysis of catch-up and convergence and global inequality. These are the topics pursued in the next two sections.

6. Convergence in real per capita incomes

In this section we examine empirical evidence for convergence in real per capita incomes at two different levels. First, convergence in incomes of people in the whole world is considered. Second, we focus on more conventional analysis of convergence in real per capita incomes of countries of the world.

5.1 ***Convergence of incomes of people in the world as a whole***

Here all the people of the world are considered as a single group and the question is whether incomes of these individuals converge over time. Let x_{jt} represent income of *j-th* individual in time period *t*. Let *M* denote the size of the population. These incomes are all expressed in real terms after adjusting for differences in price levels across countries where they belong (and also converted from different currency units into a common currency unit) and also over time. Then convergence is said to be achieved if the ratio of incomes of individuals, say *j and k*, $x_{jt}/x_{kt} \to 1 \; as \, t \to \infty$ for all j and k.

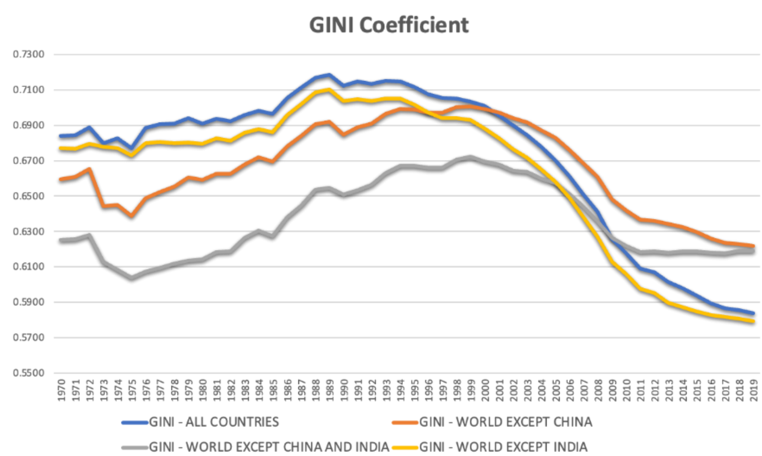
Following discussion in Section 3, This means that complete convergence in incomes of a group of people (within a country or the world as a whole) is achieved if and only if inequality in the distribution of incomes disappears, or the distribution collapses to a single point, the average income.

Testing for convergence of incomes of all people in the world is not feasible as such individual-specific real income data are not easily available. However, recent developments in modelling and fitting parametric income distributions (Chotikapanich et al 1997; Milanovic, 2002; Chotikapanich et al. 2007; Hajargasht et al 2012; and Chotikapanich et al, 2022) has made it possible to construct global income distribution which in turn make it possible to study inequality in the distribution of incomes of people in the world as whole.

The UQICD provides estimates of parameters of four parametric income distributions of which we base our analysis on the Pareto-lognormal distribution. The world income distribution is constructed as a population weighted mixture of income distributions of the whole world – resulting distribution provides a basis for measuring convergence through inequality measures.

Figure 5 presents estimates of Gini coefficients for the whole world for all the years 1970 to 2019. To disentangle the influence of China and India.

Figure 5: Inequality in the Global Distribution of Income

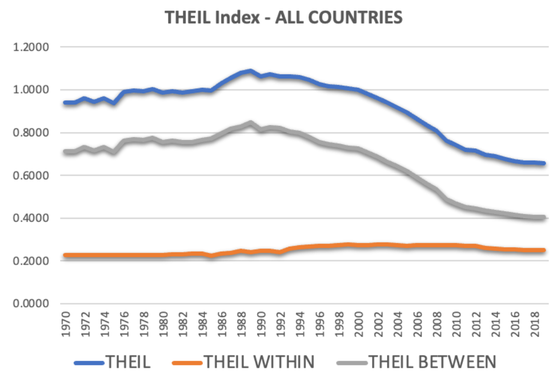


Source: UQICD Version 3.0 Module 4

The figure shows that there has been a steady increase, and hence divergence in incomes, in inequality to a peak of 0.72 in 1988 and since then has been steadily declining to 0.585 by 2019. This decline indicates a level of convergence but a long way from complete convergence. Convergence achieved since 1990's is heavily influence by strong growth performance in China and India, but China in particular. If we consider a world without China and India, we observe a slight increase in inequality since 2010. An interesting feature of the graph is that world inequality would have been lower if India were not included – a contrasting result to what happens when China is not included.

The main sources of convergence trends in inequality in incomes of people of the world shown in Figure 3 can be further examined using Theil index which is additively decomposable. Figure 6 shows the Theil index and its decomposition into within and between country inequality.

Figure 6: Theil Index of Inequality in Global Distribution of Income and its decomposition



Source: UQICD V3.0 Module 4

Trends in global inequality in Figure 6 (blue line) are similar to those shown in Figure 5 – a general increase in inequality until 1988 and a steady decline then. The main driver of global inequality in Figure 6 is the between inequality component, this component measures inequality in real per capita incomes in different countries after accounting for differences in the size of the population. Decline in between country inequality is a sign of convergence in per capita incomes of countries which will be considered in detail in the next section. A slight increase within country inequality is evident since early 1990's indicating that inequality in income distributions within countries has been increasing – this phenomenon is attributed to the effects of globalization and the likely winners and losers.

5.2 Empirical Evidence on convergence

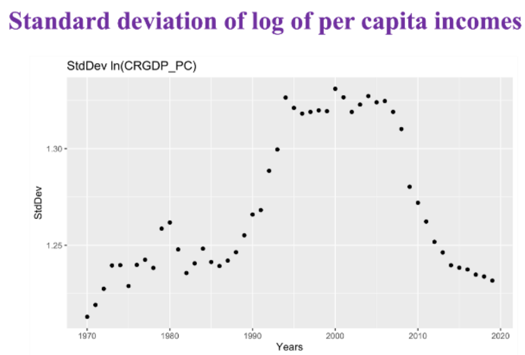
Past research on catch-up and income convergence contributed to our understanding of the dynamics of economic growth and income distribution across countries. The concept of catch-up refers to the process by which poorer countries grow faster than richer ones, thereby reducing the income gap between them. This idea is closely linked, but not identical, to income convergence, which states that over time, the incomes of countries or regions tend to converge to a similar level. With foundations from the Slow (1956) growth model, absolute β-convergence occurs when, over a specific period, relatively poor countries (or regions) grow faster than wealthier ones within a group of economies. This concept is rooted in the neoclassical growth model, which predicts that if economies differ only in their initial levels of capital and income per capita, they will eventually converge to the same steady-state level, with poorer economies experiencing faster growth. In contrast, $\sigma - convergence$ looks at inequalities in per capita incomes among countries or regions and as such examines whether the dispersion in observed per capita incomes is shrinking over time.

β-convergence can be associated with σ-convergence. Lack of β-convergence (the initially rich economy grows faster) is associated with the lack of σ-convergence (the distance between economies increases over time). Hence, it would appear that the two concepts are identical. However, at least theoretically, it is possible for initially poor countries to grow faster than initially rich ones, without observing that the cross-sectional dispersion fall over time. That is, we could find, β-convergence without finding σ-convergence (Sala-i-Martin, 1996; Barro and Sala-i-Martin (1997))

Graphical evidence of $\sigma - convergence$

Figure 7 below shows the standard deviation of log per capita incomes of different countries for the whole period. $\sigma - convergence$ is evident if there is a steady decline in the standard deviation.

Figure 7: Standard Deviation of log per capita real incomes



The graph shows increasing divergence in standard deviation of log per capita incomes until mid-1990's followed by somewhat stable standard deviation until a significant decline is present from 2005 onwards. This result is consistent with the trends in between country inequality shown in Figure 6. So, the general conclusion is that there has been a significant pattern of $\sigma -$convergence over the last two decades.

β-convergence

Following the standard literature (Sala-i-Martin (1996); Barro and Sala-i-Martin (1997)), we estimate the regression:

$$y = \alpha - (1 - \exp(\beta T))\left(\frac{1}{T}\right)x + \text{Controls} + u$$

$$y = \left(\frac{1}{T}\right)\ln\left(\frac{\text{CRGDP\_PC}_t}{\text{CRGDP\_PC}_0}\right)$$

$$x = \ln(\text{CRGDP\_PC}_0)$$

where CRGDP_PC is the constant price real GDP per capita. The dependent variable is a vector on N countries, the "x" variable is the log of CRGDP_PC at the initial time period, "0"

Relative Transition Coefficient and Convergence of Incomes

Phillips and Sul (2009) proposed a "Log t" convergence test. The test works with the relative transition coefficient, which Phillips and Sul (2009) define as follows:

$$h_{it} = \frac{\log(\text{CRGDP\_PC}_{it})}{N^{-1}\sum_i^N log(\text{CRGDP\_PC}_{it})} \quad (3)$$

The h_{it} traces out an individual trajectory for each *i* relative to the average, so h_{it} is called the 'relative transition path.' At the same time, h_{it} measures economy *i*'s relative departure from the common steady-state growth path μ_t. Thus, any divergences from the common growth component, μ_t, are reflected in the transition paths h_{it}. The transition path can be used to measure the extent of the divergent behaviour and to assess whether or not the divergence is transient. There could be a common (limiting) transition behaviour across economies $\mathrm{h_{it}} = \mathrm{h_t}$ across *i*. However, ultimate growth convergence is given by

$$\mathrm{h}_{it} \rightarrow 1 \text{for all } I, \text{ as } \mathrm{t} \rightarrow \infty.$$

As countries tend to be of different of population sizes, a more meaningful convergence measure would use a ratio relative to population weighted average of per capita incomes from all the countries. That is the alternative weighted transition coefficient is given by:

$$h_{it}^{*} = \frac{\log(\mathrm{CRGDP\_PC}_{it})}{\sum_{i=1}^{N} w_{it} \, log(\mathrm{CRGDP\_PC}_{it})} \rightarrow 1 \quad \text{as} \quad t \rightarrow \infty. \qquad (4)$$

where w_{it} is share of population of country i in the world.

Econometric testing for weak $\sigma -$ **convergence**

In this paper we implement recently introduced test for weak $\sigma -$convergence proposed in Kong et al (2019) which builds on earlier work of Phillips and Sul (2007 and 2009). Before describing the test procedure in Kong et al (2019), we present the fundamental idea of *relative transition coefficient* proposed in Phillips and Sul (2009).

The relative transition coefficients for a country *i* at year *t* is defined as:

$$h_{\{it\}} = \frac{x_{it}}{(1/N)\sum_{i=1}^{n} x_{it}} = \frac{x_{it}}{\overline{x}_{it}} \quad where \ x_{it} = \ln(CRGDP\_PC_{it})$$

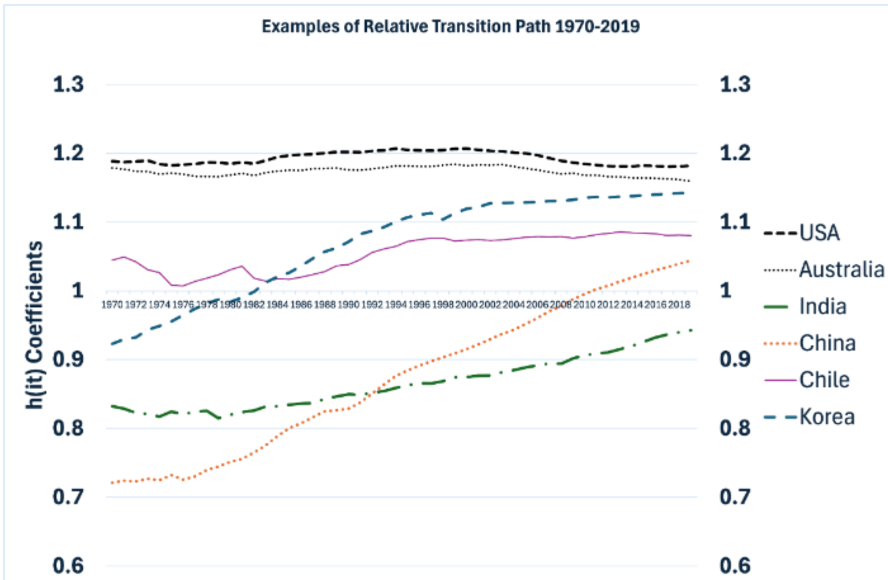
Here CRGDP_PC represents per capita real GDP expressed in 2011 prices.

The variable h_{it} traces out an individual trajectory for each *i* relative to the average, so we call h_{it} the 'relative transition path.' At the same time, h_{it} measures economy *i*'s relative departure from the common steady-state growth path μ_t. Thus, any divergences from μ_t are reflected in the transition paths h_{it}. The transition path can be used to measure the extent of the divergent behaviour and to assess whether the divergence is transient. There could be a common (limiting) transition behaviour across economies $\mathrm{h_{\{it\}}} = \mathrm{h_t}$ across *i*. However, ultimate growth convergence is given by

$$\mathrm{h_{\{it\}}} \rightarrow 1 \text{for all } i, \text{ as } \mathrm{t} \rightarrow \infty.$$

The relative transition coefficients can be computed and charted for each country over the study period.

Figure 8: Relative transition coefficients Based on Data for 159 countries: 1970-2019 (for selected countries)



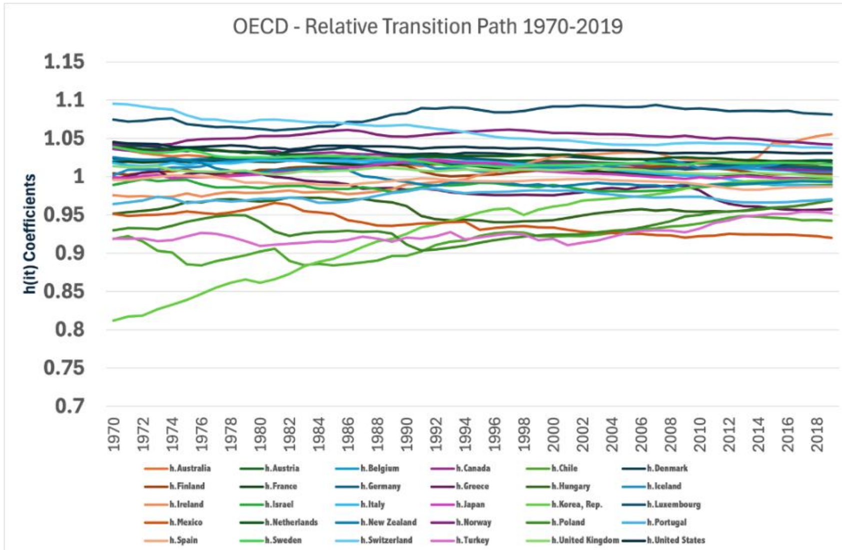
The figure shows that there is general convergence in the paths of relative transition coefficients. Interesting features to note that coefficients for Korea reached levels above 1 around 1980s whereas coefficients for China crossed unity level around 2010. In comparison, these coefficients remain below 1 and that coefficients for USA and Australia show a slight declining trend. This means that China's per capita real GDP (in logs) exceeded that the world average of log per capita real incomes by the end of the study period which means that China's growth in real per capita GDP is in excess of growth in world real GDP.

As countries tend to be of different of population sizes, a more meaningful convergence measure would use a ratio relative to population weighted average of per capita incomes from all the countries. That is the alternative weighted transition coefficient is given by:

$$h_{it}^{*} = \frac{\log(\text{CRGDP\_PC}_{it})}{\sum_{i=1}^{N} w_{it}\ log(\text{CRGDP\_PC}_{it})} \rightarrow 1 \quad \text{as} \quad t \rightarrow \infty. \qquad (5)$$

where w_{it} is share of population of country i in the world.

Figure 9: Relative transition coefficients Based on Data for 30 OCED countries: 1970-2019



In contrast to the paths of transition coefficients in Figure 8, paths of OECD countries appear to converge as they bunch-up closer to a value of 1 indicating that these countries are converging over time.

The Phillips and Sul (2009) "Log t" test is implemented by estimating the following regression

$$log\frac{H_1}{H_t} - 2\log(logt) = a + \gamma logt + u_t, for\ t = T_0, \dots, T \qquad (6)$$

where

$H_t = N^{-1}\sum_{i=1}^{N}(h_{it}^{v} - 1)^2$ and $h_{it}^{v} = h_{it}, h_{it}^{*}$ were defined in (1) and (2). The initial observation, $T_0 = rT$ for some $r > 0$. Thus the empirical log *t* regressions are based on time series data in which the first $r\%$ of the data is discarded.

We are interested in the sign of γ.

- *(i)* If $\gamma \geq 2$ (and the growth component μ_t follows a random walk with drift or a trend stationary process), then large values of γ imply *convergence in level per capita incomes*.
- (ii) If $2 > \gamma \geq 0$ the speed of convergence corresponds to *conditional convergence*, i.e. income growth rates converge over time.
- (iii) If $\gamma < 0$ there is no evidence of convergence.

Equation (2) is estimated by least squares with HAC standard errors.

Weak σ-convergence

Kong et al (2019) introduced a concept of *weak σ-convergence*, whereby cross section variation in the panel decreases over time. The paper formalizes this concept and proposes a simple-to-implement linear trend regression test of the null of no σ-convergence.

The test is based on running the regression:

$$K_{nt} = \widehat{a_{nT}} + \widehat{\phi_{nT}}\, t + \widehat{u_t}, \quad t = 1, \ldots, T$$

where,

K_{nt} cross-section sample variation, $n = 1, \ldots, N$

$\hat{u}_t$ Is the fitted residual

$\widehat{\phi_{nT}}$ is the time trend slope coefficient

This regression enables us to test the key defining property of weak σ-convergence. In particular, according to the definition, if $\underset{n\to\infty}{\text{plim}}\, K_{nt}$ exists and K_{nt} is a decreasing function of *t*, then weak σ-convergence holds. In this event, in terms of the regression, we expect the slope coefficient to be significantly negative, whereas if it is not significantly different from zero or is greater than zero, then the null of no σ-convergence cannot be rejected.

Under the null, H_0: no convergence, a t-ratio, $t_{\widehat{\phi_{nT}}}$, is computed with an estimated long-run variance computed using the residuals with a Bartlett–Newey–West estimator (see equation (24) of Kong et al (2019) (page 193) for details.

This is a one-sided test with a critical value of -1.65 at the 5% significance level.

Empirical results on convergence

Table 5: Absolute β-Convergence Regressions – (World)

	1975-85	1985-1995	1995-2005	2005-19	1975-2019
Estimated β	0.025***	0.022***	0.020***	0.021***	0.019***
	(0.003)	(0.004)	(0.004)	(0.003)	(0.001)
N	159	159	159	159	159
R2	0.434	0.333	0.177	0.319	0.702
AIC	-717.9	-690.3	-723.2	-797.0	-1017.9
BIC	-699.5	-671.9	-704.8	-778.5	-999.5

Table 5: Absolute β-Convergence Regressions – (World)					
	1975-85	1985-1995	1995-2005	2005-19	1975-2019
Log.Lik.	364.966	351.147	367.590	404.477	514.951
F	29.508	19.193	8.272	18.021	90.774
RMSE	0.02	0.03	0.02	0.02	0.01

Regression: y= a – βx + FE + noise; if β > 0 the data exhibits absolute convergence (Sala-i-Martin, 1996, Econ Journal)
y = (1/(T-t))*ln(CRGDP_PC$_T$/CRGDP_PC$_t$); x = ln(CRGDP_PC$_t$); WB income group fixed effects.

Table 6: Absolute β-Convergence Regressions (OECD)			
	1975-85	1995-2005	2005-19
Estimated β	0.009**	0.021***	0.017***
	(0.004)	(0.004)	(0.004)
N	30	36	36
R2	0.138	0.438	0.367
AIC	-180	-210.1	-229.6
BIC	-175.8	-205.4	-224.9
Log.Lik.	92.994	108.074	117.801
F	4.501	26.549	19.751
RMSE	0.01	0.01	0.01

Regression: y= a – βx; if β > 0 the data exhibits absolute convergence (Sala-i-Martin, 1996, Econ Journal).

y = (1/(T-t))*ln(CRGDP_PC$_T$/CRGDP_PC$_t$); x = ln(CRGDP_PC$_t$);

Table 7: Phillip and Sul (2009) Convergence Test

Transition Parameter	Estimate	World	OECD	Asia	SSF	LMI	MEA	UMI
Unweighted (h_{it})	$\hat{\gamma}$	-0.675***	-0.349***	-0.347***	-0.899***	-0.234***	-0.494***	**0.232*****
Weighted (h_{it}^*)	$\hat{\gamma}$	-0.420***	-0.351***	-0.076	-0.920***	**0.1195***	-0.361***	**1.321*****
N		159	36	21	45	46	16	49
rT		45	45	45	45	45	45	45
Conclusion		No C	No C	No C	No C	Mixed	No C	Cond. C

*** Significant at 1% level. ** Significant at 5% level, * Significant at 10% level

$0 < \hat{\gamma} \leq 2$ the speed of convergence corresponds to conditional convergence, i.e. income growth rates converge over time.

Table 8: Kong et al (2019) Weak σ-convergence test

	World	OECD	Asia	SSF	LMI	MEA	UMI
t-stat	**-11.504**	370.446	89.700	266.503	127.087	**-134.582**	**-134.460**
Conclusion	Weak σ -C	No evidence of weak σ -C				Weak σ -C	Weak σ -C

Note: One sided t-test: Critical value -165. Negative implies evidence of weak σ-convergence

Discussion

The results for the World (here represented by 159 countries that existed over the period 1970-2019) is that there is evidence of β-convergence (Table 5). The finding is consistent for sub-periods as well as for the whole time period of the sample. The evidence is also strong for the OECD, especially for time periods after 1995 (Table 6).

There is no evidence of convergence in levels per capita income for the world or any of the sub-groupings of countries that we have tested using the Log *t* test (Kong et al (2019)) as shown on Table 8. There is some evidence for *conditional convergence* for the Upper Middle Income group-UMI (as defined by World Bank in 2019) and mixed evidence for the Lower Middle Income group-LMI, where conditional convergence is found when using the transition parameter computed using population shares weights (see Equation (2)). The evidence however, is only at the 10% significance level. It is important to note that the time series we are using are short and as shown by the Monte Carlo evidence in Table 5 of Kong et la (2019), the test does not have the correct size and lacks power for these small samples.

When applying the t-test for weak σ-convergence of Phillips and Sul (2009), we find evidence for the World, the Upper Middle Income group-UMI and the Middle East and North Africa group-MEA. Phillips and Sul (2009)'s Table 5 computed the test for 152 countries in the PWT (period 1970-2003) and found the estimate of $\hat{\gamma}$ to be -0.88 and significant at the 5%. This estimate is consistent with ours, as we find it to be -0.675 and significant at the 1% level (see Table 7). It is also significant using the weighted transition parameter from Equation (2). For the OECD we do not have an equivalent period or set of countries that were included in their OECD estimates; however, our findings are consistent with theirs. They mostly found non-significant estimates, except for the 1870-1929 period when the estimate is negative and significant (-0.42) indicating no convergence. Our estimates are -0.35 whether we use Equation (1) or Equation (2) to compute the transition parameter.

7. Convergence in Income Distributions

In the last two decades, research on global inequality focused on studying trends in inequality not only at the country level but also at the global level by combining income distributions in different countries. Milanovic (2002) pioneered work in this direction with further work reported in Warner et al (2014) and Milanovic (2024). Main findings from these studies are that there has been a general decline in global inequality, largely driven by reductions in between-country inequality. Contribution of fast growing and populous countries like China and India are acknowledged. A recent phenomenon is a rise in within-country inequality in different regions. There is some evidence of polarization of incomes demonstrated through the elephant curve shaped growth incidence curves. The evidence based on income inequality literature is mixed in terms of convergence of incomes. These studies are primarily focused on overall inequality measures that account for distributional characteristics within countries but do not consider the evolution and convergence of income distributions across countries.

As convergence in income distributions a relatively new notion that focuses on the evolution of income distributions, at the country level as well as at regional or global level, over time, new methods and tools for measuring closeness (or divergence) between distributions are necessary. In this paper we elaborate on this concept and propose new measures of divergence between pairs and groups of income distributions. These measures are then empirically implemented using data drawn from UQICD Version 3.0.

7.1 How has the distribution of income changed within a subpopulation over time?

A good starting point for answering this question is by defining an appropriate divergence measure between two distributions.

Divergence between two distributions: An important class to consider is the α-divergence[3] between two distributions $q_1(x)$ and $q_2(x)$ defined in the following way (see e.g Cowell et al. 2009)

$$D_\alpha(q_1, q_2) = \frac{1}{\alpha(\alpha-1)} \int \left(1 - q_1(x)^\alpha q_2(x)^{1-\alpha}\right) dx \qquad \alpha \neq 1,0$$

$$D_1(q_1, q_2) = K(q_1, q_2) = \int q_1(x) \ln\left[q_1(x)/q_2(x)\right] dx \qquad \alpha = 1$$

These Divergence measures are scale dependent. In many situations, we are interested in distributional changes that are scale independent. A relative Theil-entropy can be defined as (e.g. Cowell et al. 2009)

$$T_\alpha(q_1, q_2) = \frac{1}{\alpha(\alpha-1)} \int_0^1 [s_1(p)^\alpha s_2(p)^{\alpha-1} - 1]\, dp \qquad \alpha \neq 0,1$$

---

[3] One could start with even a more general divergence measure called *f*-divergence defined as $D_f(q_1, q_2) = \int q_1(x) f\left(q_1(x)/q_2(x)\right) dx$ where f is any convex function from $(0, \infty) \to R$ with $f(1) = 0,\ f'(1) = 0,\ f''(1) = 1$.

$$T_1(q_1,q_2) = \int_0^1 s_1(p)\ln\left[s_1(p)/s_2(p)\right]dp$$

where π denotes population size normalized to [0,1] and $s(\pi)$ is infinitesimal share of income (i.e. those located in neighbourhood around the p-th quantile have a share $s(p)dp$ in total income). $s(p)$ has the same properties as a density function i.e. $s(p) \geq 0$ and $\int_0^1 s(p)dp = 1$. The celebrated Theil index is a special case of this measure with $s_2(\pi) = 1$ and defined as $T_1 = \int_0^1 s_1(\pi)\ln s_1(\pi)d\pi$ i.e. the Theil index measures the divergence of a given distribution from the egalitarian distribution.

Note that using $s_1(p) = \dfrac{F_1^{-1}(p)}{\int_0^1 F_1^{-1}(p)} = \dfrac{x}{m} \Rightarrow \begin{cases} p = s_1^{-1}[x/m] \Rightarrow dp = q_1(x) \\ s_2(p) = F_2^{-1}(F_1(x)) \end{cases}$

we can write

$$T_a(q_1,q_2) = \frac{1}{a(a-1)}\left[\int_0^\infty \left(\frac{x}{m_1}\right)^a \left(\frac{F_2^{-1}(F_1(x))}{m_2}\right)^{1-a} q_1(x)dx - 1\right]$$

$$T_1(q_1,q_2) = \int_0^\infty \frac{x}{m_1}\ln\left[\frac{x/m_1}{F_2^{-1}(F_1(x))/m_2}\right]q_1(x)dx$$

This measure has been studied by Cowell et al. (2009, 2013) within a nonparametric framework and Hajargasht (2024) for parametric distributions. Hajargasht (2024) has derived closed form formulas for this measure for some of the well-known distributions. In particular

$$\text{if } q_1(x) = \text{lognorm}(m_1, s_1^2) \,\&\, q_2(x) = \text{lognorm}(m_2, s_2^2) \Rightarrow T_1(q_1,q_2) = (s_1 - s_2)^2/2$$

$$\text{if } q_1(x) = \text{Pareto}(a_1) \,\&\, q_2(x) = \text{Pareto}(a_2) \Rightarrow T_1(q_1,q_2) = \ln\frac{(1-1/a_1)}{(1-1/a_2)} + \frac{a_2 - a_1}{a_2(a_1 - 1)}$$

Given a sample of observations from subpopulations, one can estimate parametric distributions and the covariance matrix for the parameters. We can then use the delta method to obtain the standard error or confidence interval for the measure. For other distributions such as Pareto-lognormal, there is no closed form solution, but one can calculate the measures by using numerical integration.

If one has grouped data with n groups and where s_i is the share of income group. Assuming a linearly interpolating Lorenz curve, it can be shown that (see appendix)

$$T_a = \frac{1}{a(a-1)}\sum_{i=1}^{n}\left(s_{1i}^a s_{2i}^{1-a} - 1\right)$$

$$T_1 = \sum_{i=1}^{n} s_{1i}\ln\left(s_{1i}/s_{2i}\right) \tag{7}$$

Distributional change within a population over time: To measure distributional change over time, we propose to apply the above criteria to a population across time each period with respect to previous period. As an example, to explore if the the US income distribution have changed over the last fifty years, Hajargasht (2024) has computed the T_1 -measure of divergence for US at 10-year intervals: 1970, 1980, 1990, 2000, 2010, and 2019. To check robustness, the lognormal, Dagum, and GB2 distributions are employed. The parameter estimates and their standard errors for the lognormal and GB2 are obtained from UQICD version 3 (see Rao et al., 2022); the parameters for the Dagum distribution are estimated using income share data from UQICD.[4]

Table 9: Estimates of Income Distributions and Divergence Measures for US

Year	**Lognormal**			**Dagum**				**GB2**				
	s	***Theil***	***100T***	***P***	***a***	***Theil***	***100T***	***P***	***q***	***a***	***Theil***	***100T***
1970	0.690	0.238		0.581	3.246	0.2264		0.633	1.124	3.026	0.2220	
	0.003	0.0024		0.013	0.041	0.0035		0.041	0.088	0.152	0.0040	
1980	0.673	0.2265	0.0142	0.647	3.123	0.2307	0.0134	1.168	2.262	1.982	0.2065	0.0790
	0.003	0.0023	0.0082	0.015	0.039	0.0037	0.0104	0.100	0.252	0.118	0.0029	0.2247
1990	0.738	0.2723	0.2112	0.642	2.850	0.2808	0.2325	1.375	2.850	1.606	0.2441	0.1675
	0.004	0.0028	0.0328	0.015	0.036	0.0048	0.0547	0.133	0.374	0.106	0.0034	0.4078
2000	0.756	0.2861	0.0167	0.744	2.658	0.3049	0.0676	0.835	1.148	2.433	0.2953	0.4594
	0.004	0.0029	0.0098	0.019	0.033	0.0057	0.0356	0.060	0.092	0.126	0.0073	0.2787
2010	0.775	0.3003	0.0172	0.679	2.658	0.3177	0.0208	0.951	1.584	2.041	0.2863	0.0800
	0.004	0.003	0.0102	0.017	0.034	0.0059	0.0118	0.072	0.143	0.110	0.0055	0.0874
2019	0.792	0.3131	0.0139	0.739	2.520	0.3444	0.0567	0.961	1.407	2.058	0.3149	0.0842
	0.004	0.0032	0.0093	0.019	0.032	0.0068	0.0373	0.073	0.122	0.110	0.0073	0.0561

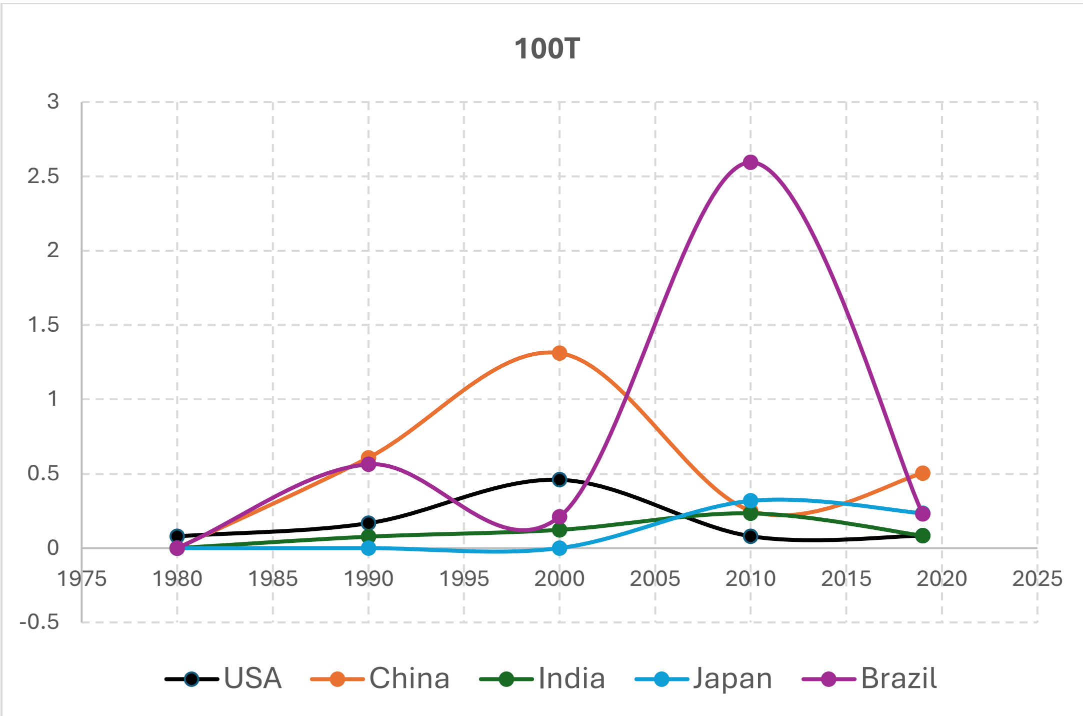
Notes: Parameter estimates, standard errors and income share data are sourced from UQICD. The scale parameter is excluded. The number under each value is the standard error. As *H* values are small, these are multiplied by 100.

Table 9 reports the Theil index and the *H*-measure. The Theil index shows a steady increase in inequality in US over the years, with an exception for 1980 when the lognormal distribution is used. The trend in inequality reported here is in line with a generally observed increase in inequality in US over the study period. The estimated *H* values show an interesting pattern. The income distribution in US remains stable over the period 1970 to 1980 and post 2000. From 1980 to 1990 and possibly from 1990 to 2000s, the divergence measure suggests a substantial shift in the income distribution. A similar trend is exhibited by all three distributions except for those from the GB2 distribution for 1990-2000.

The following graph shows the distributional change for several countries including USA, China, India, Japan and Brazil. Again, each point shows the distributional change with respect to the previous decade. According to the graph, distributional change has been much pronounced in Brazil from 2000 to 2010 period and China from 1980 to 2000.

[4] Results for the Dagum distribution are available from the authors upon request.

Figure 10: Divergence in Country-specific Income Distributions



Source: Hajargasht (2024); The measures are based on parametric distributions based on data sourced from UQICD.

6.2 How different is income distribution between a set of subpopulations at a particular time?

This question compares income distributions across different subpopulations at a single point in time. The goal is to understand the disparities or similarities between subpopulations. This requires defining divergence between distributions.

Divergence between multiple distributions: One natural extension of the measure to multiple distributions is by averaging of all bivariate distances i.e.

$$\Delta_J(q_1,\ldots,q_M) = \frac{1}{M(M-1)}\sum_{i=1}^{M}\sum_{j=1}^{M} D(q_i,q_j)$$

If sub-populations have different population weights and we want to take the weights into account, the measure can be defined as

$$\Delta_{wJ}(q_1,\ldots,q_M) = \frac{1}{1-\sum_{i=1}^{M} w_i^2}\sum_{i=1}^{M}\sum_{j=1}^{M} w_i w_j D(q_i,q_j)$$

Another method measures the average divergence of each distribution from the average of all distributions.

$$\Delta_{IR}(q_1,\ldots,q_M) = \frac{1}{M}\sum_{i=1}^{M} D\left(q_i,\frac{1}{M}\sum_{i=1}^{M} q_i\right)$$

There is an extension of this that computes the average divergence of each distribution from the barycentre (a weighted average) of all distributions. This measure is often known as the Jensen-Shannon Dissimilarity (JSD) measure.

$$\Delta_{JSD_1}(q_1,...,q_M) = \sum_{i=1}^{M} w_i D\left(q_i, \sum_{i=1}^{M} w_i q_i\right)$$

JSD index satisfies several appealing properties that makes it particularly useful in various applications.

- *(i)* JSD is symmetric (i.e. it is independent of the order of distributions)
- *(ii)* It is non-negative and bounded e.g. $0 \le \Delta_{JSD,1} \le \log M$
- *(iii)* It equals zero if and only if all distributions being compared are identical i.e. $H_{JSD}(q_1,...,q_M) = 0 \Leftrightarrow q_1 = ... = q_M$
- *(iv)* The square root of the JSD, often referred to as the Jensen-Shannon distance, is a proper distance measure as it satisfies triangular inequality meaning that it quantifies the divergence between distributions in a manner akin to the Euclidean distance in geometry i.e. $\sqrt{\Delta_{JSD}(q_1,q_2)} \le \sqrt{\Delta_{JSD}(q_1,q_3)} + \sqrt{\Delta_{JSD}(q_3,q_2)}$

Distributional Gini and Distributional CV: Anderson et al. (2017, 2021) introduces two similar measures that they call distributional CV (DCV) and distributional Gini (DisGini) as follows

$$DCV = \frac{1}{1-\sum_{j=1}^{N} w_j^2} \sum_{j=1}^{M} w_j D\left(q_j, \sum_{j=1}^{M} w_j q_j\right)$$

$$DisGini = \frac{1}{1-\sum_{j=1}^{N} w_j^2} \sum_{i=1}^{M} \sum_{j=1}^{M} w_i w_j D(q_i, q_j)$$

but they use total variation as their measure of distance i.e.

$$D(q_1,q_2) = TV(q_1,q_2) = \frac{1}{2} \int |q_1(x) - q_2(x)| dx .$$

***Scale-free Divergence between Distributions*:** The above measures can be extended to scale-free version as follows

$$\mathrm{T}_J(q_1,...,q_M) = \frac{1}{1-\sum_{i=1}^{M} w_i^2} \sum_{i=1}^{M} \sum_{j=1}^{M} w_i w_j T(q_i,q_j)$$

$$\mathrm{T}_{JSD_1}(q_1,...,q_M) = \sum_{j=1}^{M} w_j T\left(q_j, \bar{q}\right) \text{ where } \bar{q} = \sum_{j=1}^{M} w_j q_j$$

It can be shown that the second measure is not scale-invariant. Another version that can be scale-invariant can be defined as

$$\Upsilon_{JSD_2}(q_1,...,q_M) = \sum_{j=1}^{M} w_j T\left(q_j, \breve{q}\right) \text{ where } \breve{q} \text{ is such that } \breve{s} = \sum_{j=1}^{M} w_j s_j$$

For the rest of the analysis, we use the Jeffery's $\Upsilon_J(q_1,...,q_M)$. Using the results from the previous section we can compute this measure for parametric distributions. For example, assuming lognormal distributions for each subpopulation we obtain:

$$\Upsilon_J^1(q_1,...,q_M) = \frac{1}{1-\sum_{j=1}^{M} w_j^2} \sum_{i=1}^{M} \sum_{j=1}^{M} \frac{1}{2} w_i w_j (s_i - s_j)^2$$

This measure can also be estimated with grouped data using the group data. Below we have computed the divergence measure between countries within 9 regions in the World using a Pareto-lognormal distribution.

Table 10: Divergence measures for Groups of Countries, 1970 to 2019 at 10-year intervals

	Reg1	**Reg2**	**Reg3**	**Reg4**	**Reg5**	**Reg6**	**Reg7**	**Reg8**	**Reg9**
1970	0.81474	0.88686	3.8072	0.15086	1.1119	1.9502	0.95191	2.0429	1.0409
1980	0.81496	0.8869	3.8074	0.14339	1.138	1.9441	0.95232	1.9758	1.0693
1990	0.64511	0.77874	3.9987	1.1957	1.7554	3.1991	0.68744	2.0199	0.9329
2000	0.49891	0.94338	1.9321	0.91208	0.60588	2.6355	0.72925	1.5353	0.51428
2010	0.52046	0.54569	1.7586	1.1928	0.66956	1.1845	1.2627	1.0458	0.53641
2019	0.35624	0.37127	2.5184	1.4541	0.81457	1.6684	1.0099	0.97413	0.75129

Reg1=["BGD";"BTN";"IND";"LKA";"MDV";"NPL";"PAK"];

Reg2=["AUS";"CHN";"FJI";"IDN";"JPN";"KIR";"KOR";"LAO";"MNG";"MMR";"MYS";"PHL";"PNG";"SLB";"THA";"TON";"VUT";"VNM"];

Reg3=["AGO";"BDI";"BEN";"BFA";"BWA";"CAF";"CIV";"CMR";"COD";"COG";"COM";"ETH";"GAB";"GHA";"GIN";"GMB";"GNB";"KEN";"LBR";"LSO";"MDG";"MLI";"MOZ";"MRT";"MUS";"MWI";"NAM";"NER";"NGA";"RWA";"SEN";"SLE";"STP";"SDN";"SYC";"TCD";"TGO";"TZA";"UGA";"ZAF";"ZMB";"ZWE"];

Reg4=["CAN";"USA"];

Reg5=["ALB";"ARM";"AUT";"AZE";"BLR";"BEL";"BIH";"BGR";"HRV";"CYP";"CZE";"DNK";"EST";"FIN";"FRA";"GEO";"DEU";"GRC";"HUN";"ISL";"IRL";"ITA";"KAZ";"KGZ";"LVA";"LTU";"LUX";"MDA";"NLD";"NOR";"POL";"PRT";"ROU";"RUS";"SRB";"SVK";"SVN";"ESP";"SWE";"CHE";"TJK";"TUR";"UKR";"GBR";"UZB"];

Reg6=["BLZ";"BOL";"BRA";"CHL";"COL";"CRI";"DOM";"ECU";"GTM";"GUY";"HND";"HTI";"JAM";"LCA";"MEX";"NIC";"PAN";"PER";"PRY";"SLV";"TTO";"URY"];

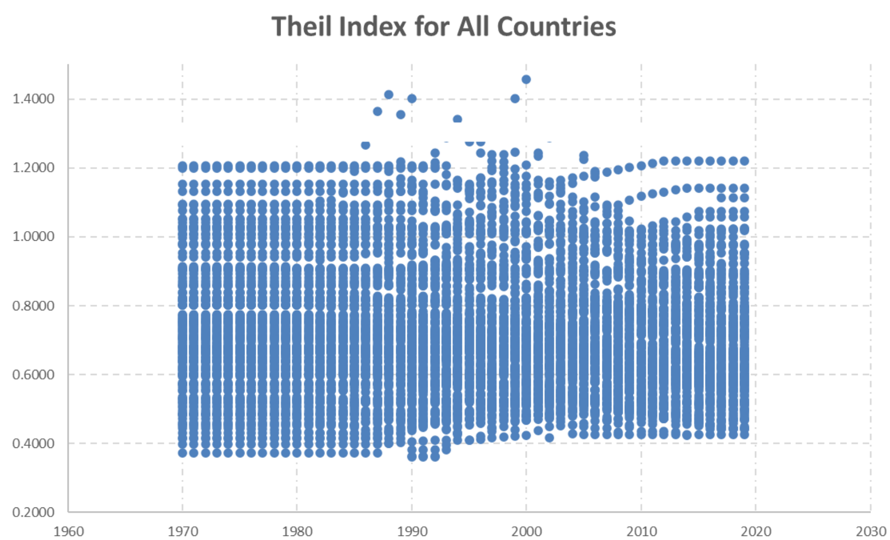
Reg7=["ARE";"DJI";"DZA";"EGY";"IRN";"IRQ";"ISR";"JOR";"LBN";"MAR";"MLT";"SYR";"TUN";"YEM"];
Reg8=["AUS";"AUT";"BEL" ;"CAN";"CHE" ;"CHL" ;"CZE" ;"DEU";"DNK" ;"ESP"; "EST";"FIN";"FRA";"GBR";"GRC";"HUN";"IRL";"ISL";"ISR";"ITA";"JPN";"KOR";"LUX";"MEX";"NLD";"NOR";"POL";"PRT";"SVK";"SVN";"SWE";"TUR";"USA"];

Reg9=["AUT";"BEL";"BGR";"CYP";"CZE";
"DEU";"DNK";"ESP";"EST";"FIN";"FRA";"GBR";"GRC";"HRV";"HUN";"IRL";"ITA";"LTU";"LUX";"LVA";"MLT";
"NLD";"POL";"PRT";"ROU";"SVK";"SVN";"SWE"];

6.3: How has the difference in income distribution among subpopulations evolved over time?

To examine the evolution of divergence between subpopulations over time, we plot the divergence measure across different periods. This measure specifically captures the divergence between income distributions, indicating either convergence or divergence in income inequalities. Before presenting this measure, we first introduce Figure 9, which displays the Theil inequality index for all countries over time. We observe a reduction in the spread of inequalities from approximately 1990 to 2010, suggesting evidence of some degree of sigma-convergence.

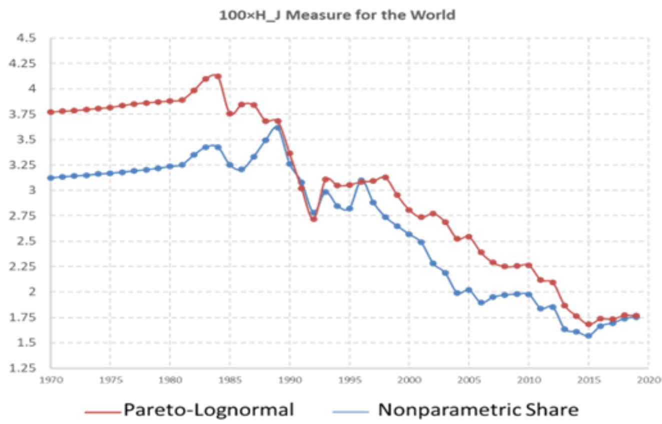
Figure 11: Dispersion of Theil indices across all countries



Source: Computations using UWQICD data

Figure 11 illustrates the J-divergence measure over time, using both the Pareto-lognormal and nonparametric share formulas.

Figure 12: Divergence in Income Distributions of Countries: 1970- 2019



Source: UQICD V3.0 and own calculations

From 1970 to 1990, there is limited evidence of convergence and a small degree of divergence. From mid-1980's there is strong evidence for convergence in distribution emerges from 1985 to around 2015. There are signs of slight divergence since 2015. Interestingly, these trends resemble the trends in global inequality measured using Gini and Theil's measures presented in figures 3 and 4. While globalization may have a role to play in the level of convergence, the exact mechanism leading to convergence in income distribution needs to be studied which is an excellent follow-up study to this paper.

8. Conclusions

Convergence in the world economy is a topic with considerable scope. Literature is full of studies on catch-up and convergence in incomes, convergence in price levels, productivity catch-up and convergence, technology transfers, convergence in factor remunerations, factor shares not to mention convergence in institutional, organizational and managerial structures across countries. However, the ultimate goal of these studies would then be to examine the economic welfare levels enjoyed by individuals and therefore study of convergence in incomes, inequality and income distributions. Consequently, the primary focus of this study is on real per capita GDP and income distributions. In this study we utilize data series available from the recently released University of Queensland International Database (UQICD) V 3.0 which is unique as it provides internationally comparable panels of real incomes, income distributions and inequality measures at the country level as well as for regional country groupings for the period 1970 to 2019. Informal analysis of real income data from UQICD as well as formal analysis using well-developed techniques to study income convergence have shown that since the beginning of 1990's there has been an appreciable convergence evidenced by significant reductions in global inequality as well as significance in tests for $\sigma-$ and $\beta-$ convergence. An interesting finding from UQICD data is that while an elephant curve, for growth incidence curve, was evident for the period 1970 to 2000, it seemed to disappear once additional data added and growth incidence is computed over the fifty-year period. A novel contribution of the paper is the development of a framework and measures to study divergence/convergence of income distributions, slightly more comprehensive than just inequality measures available in the literature. Our measure of divergence of income distributions across countries of the world has fallen significantly, once again, since 1990's. As the primary focus of the paper is on measurement and testing for income and income distribution convergence, no major attempt has been made to offer potential explanations for the findings from the study.

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